Satellite Attitude Determination with Attitude Sensors and Gyros using Steady-state Kalman Filter

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Abstract—Attitude determination, along with attitude control, is critical to functioning of every space mission. Here, we investigate the application of a steady-state Kalman Filter for attitude determination using attitude sensors (sun sensor and horizon sensors) and rate integrating gyros. The three-axis steady-state Kalman Filter formulation, adopted from literature, offers obvious computational advantages for on-board implementation on spacecrafts. Thus, closed loop simulation, involving the attitude determination formulation, true attitude dynamics and attitude control, for a satellite are developed in MATLAB®. Simulink. Based on the simulation, performance of the filter for an Earth-pointing Low Earth Orbit satellite is presented, in order to examine the applicability of the steady-state Kalman Filter for satellite attitude determination.

I. INTRODUCTION

Maintaining a desired orientation in space, with a specified level of accuracy, is a mission requirement for every spacecraft. Attitude determination along with attitude control is responsible for satisfying this requirement. The level of accuracy required varies, depending on the function of the spacecraft, which in turn effects the selection of sensors and complexity of algorithm.

During the past four decades, a lot of research has been done in the area of spacecraft attitude determination. Various algorithms exist in the literature, with varied level of complexity and applicability. The choice of algorithm for a mission depends on pointing accuracy requirements, the type of sensors available and capability of the on-board computer.

Following [1] and [6], which provide a comprehensive survey of existing attitude determination approaches, attitude determination methods can be broadly categorized as

- Single Frame Methods
- Sequential Methods

Single frame methods are essentially those methods which do not require any a priori information/estimate and give information of attitude based only on current measurements. These methods require a minimum of two independent vector measurements, so as to determine the complete attitude information. Sensors measuring reference vectors, such as, Sun sensors, Horizon sensors and Star trackers, are used to determine attitude using single frame methods. Methods also exist which optimally utilize information from more than two reference vectors to provide the attitude information. Some examples of single frame methods include, TRIAD, QuEst (Quaternion Estimator) and Davenport’s q-method.

Sequential methods are those which utilize past information and provide the solution of the attitude using both the past estimate and current measurement. This mixing of past estimate with the current measurement is usually achieved by employing filters (such as, Kalman Filter). These methods are generally more accurate and robust than single frame methods and can also provide reliable attitude information when some measurements are missing. Furthermore, sequential methods being recursive have minimum memory requirements. Correction or updates to the estimate can be achieved through any of the reference sensors used for single frame methods.

Here, we present simulation for a sequential attitude determination algorithm using attitude measurements (from Sun sensors and Horizon sensors) and gyros. Application of Kalman Filter for these set of sensors has been explored in detail [4]. However, calculation of Kalman Gains is computationally burdensome. Hence, the performance of a steady-state Kalman Filter is examined for the above set of sensors. The three-axis Kalman Filter formulation has been adopted from [2], which develops upon the single-axis steady-state analysis of [1]. Corrections obtained from attitude measurements are assumed to be obtained from a single-frame method, and are modeled as true attitude with white noise. The rate integrating gyro output provides the incremental attitude information corrupted by varying drift-rate bias, white noise and quantization noise.

II. SENSOR MODELS

In this section, we describe the sensor models being used in our attitude determination algorithm.

A. Rate Integrating Gyros

Gyros are inertial sensors which measure the information related to angular velocity in the body frame. Gyros are of various types - such as, mechanical gyros, ring laser gyros (RLG), fiber optic gyros (FOG) - and can be classified based on their accuracy, mechanisms and form of output. Rate gyros measure angular rate directly, while the rate-integrating gyros measure integrated angular rate. Rate integrating gyros are typically more accurate [1]. For our analysis rate integrating gyros are used; also, the mathematical parameters of the gyro errors are taken corresponding to that of Fiber Optic Gyros.

A generic model of the rate integrating gyro is used ([1]), allowing for testing the estimation algorithms for different types of gyros. The rate integrating gyros measure the incremental attitude information along with some noise. The overall
measurement equation along with the errors for the gyro is given as
\[
\Delta \varphi_k = \Delta \theta_k + Th \beta_k + \nu_{q,k}
\]
where
\[
\Delta \theta_k = \int_{T_{\text{gyro}}}^{(k+1)T_{\text{gyro}}} \omega_{in}(t)dt
\]
and
\[
\omega_{in} = A_{\text{misalign}} \omega
\]

The term \(\Delta \theta_k\) expresses the true change in the spacecraft attitude, while the \(\Delta \varphi_k\) denotes the rate integrating gyro output. The gyro drift rate bias is denoted by \(\beta_k\). The gyro measurements, while zero-mean noise due to continuous time random-walk rate vector \(\nu_{u}(t)\) and drift acceleration \(\nu_{u}(t)\) is expressed by \(\beta_k\). The variance of \(\beta_k\) is a \(3 \times 3\) diagonal matrix for which the diagonal element is \(\sigma^2_3 = \sigma^2_{\text{gyro}} + \sigma^2_{\text{acc}} T_3^2/3\), where \(\sigma^2_{\text{gyro}}\) and \(\sigma^2_{\text{acc}}\) represent the power spectral densities of the scalar elements of \(\nu_{u}\) and \(\nu_{a}\), respectively. The gyro drift-rate bias changes and this change is represented as
\[
b_k = b_{k-1} + \nu_{q,k}
\]
where \(\nu_{q,k}\) is a zero-mean discrete random-rate noise vector, with variance of each element being \(\sigma^2_q = \sigma^2_{\text{gyro}} T_3\) ([11]). The term \(\nu_{q,k}\) represents the quantization error of the gyro. Lastly, scale and misalignment errors occur due to the mechanical misalignments in the system, or intrinsic sensor errors. These errors are quantified using the misalignment matrix, which describes the transformation between the expected and actual sensor axes,
\[
A_{\text{misalign}} = \begin{bmatrix}
S_x & \delta_{xy} & -\delta_{xz} \\
-\delta_{yx} & S_y & \delta_{yz} \\
\delta_{zx} & -\delta_{zy} & S_z
\end{bmatrix}
\]

For simulations presented later, we have used the parameters of a rate integrating type Fiber Optic Gyro; the same are listed in Table I. No misalignment or scale errors are considered. The sampling rate of the gyros for the simulation is taken as 100 Hz.

### Table I - Gyro Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_{\text{u}})</td>
<td>7.27</td>
<td>(\mu\text{rad} / \sqrt{\text{s}})</td>
</tr>
<tr>
<td>(\sigma_{\text{n}})</td>
<td>(3 \times 10^{-4})</td>
<td>(\text{rad} / \sqrt{\text{s}^2})</td>
</tr>
<tr>
<td>(T_{\text{gyro}})</td>
<td>0.01</td>
<td>s</td>
</tr>
</tbody>
</table>

### B. Attitude Sensors

Attitude sensors provide information of total attitude, as opposed to gyros which provide information about the incremental attitude. These sensors measure a reference vector and by comparing it with its modeled vector provide the requisite attitude information. Determination of attitude from vector measurements is accomplished by the use of single frame methods. In our analysis, we assume the vector measurements have been processed, and the output of attitude sensors is described directly as the true attitude along with white noise.

Sun sensor and horizon sensors are used as the two attitude sensors. Sun sensors determine the direction of the sun (sun vector) with respect to the satellite. A reference value of the sun vector can be determined using the position of the satellite, which can be compared against the measurement to determine the yaw attitude. However, one disadvantage of using sun sensor is that no information is available during the eclipse phase of the satellite, i.e. when the satellite is in the shadow of the earth. This disadvantage is more prevalent in the Low Earth Orbits. Horizon sensors, also known as earth sensors, sense the earth’s radiation in certain frequency bands, namely, narrow infrared wavelength band corresponding to emission line of CO₂ molecule. Typically, heading and attitude are used separately, where sun sensors are used to measure heading and horizon sensors are used to measure pitch and roll. Thus along with the sun sensor, the horizon sensors provide complete attitude information.

As mentioned earlier, a simple model considering only white noise as the attitude error in the sensor output is used for the two attitude sensors. The standard deviation of the discrete white noise (\(\sigma_n\)) is different for sun sensor (yaw) and horizon sensors (roll and pitch). For simulation, the standard deviation in the discrete time white noise along the three axis is taken as.

The attitude sensors provide output at a lower sampling rate as compared to the gyros. For our simulation, they are assumed to be working at 1 Hz. Apart from the above mentioned sensors, on board hardware is required to measure the position of the spacecraft. For our analysis, we assume the position of the satellite is known perfectly, and no error models for position errors and orbit propagation are considered.

### III. Attitude Determination Formulation

A steady-state three-axis Kalman Filter adopted from ([11] and [2]) is used for attitude determination. The steady-state filter is computationally better than the recursive gain Kalman filter as the Kalman Gains are constant and can be computed beforehand. This eliminates the need of on-board matrix computations; since, calculation of Kalman gains and associated covariance propagation is not required for attitude determination.

The steady-state filter provides the estimate of the spacecraft attitude and the gyro drift-rate bias. The gyro measurements, which are available at a very high rate (100 Hz), are used as the process model for the filter. The attitude sensors are used for correction of the attitude estimate and gyro drift-rate bias and represent the measurement model. The prediction step using the rate integrating gyro measurements takes place at a higher rate, while the correction step is used only after a predetermined update interval (\(T_{\text{up}}\)).
Propagation equations, as they occur at a different rate, are described by using the subscript \( k \). At the \( n \)-th gyro interval correction step is applied using the attitude sensors. In the following analysis, the indices \((-\)
 and \((+\)) indicate the estimates prior to and post measurement updates from attitude sensors, respectively. The choice of update interval depends on the sensor error characteristics, sensor sampling rate and required pointing accuracy, and is discussed subsequently.

### A. Prediction

The errors in rate integrating gyro measurement \( \Delta\hat{\psi} \) are described in equation 1. Based on this gyro equation, the propagation equations for the filter which utilizes the gyro measurements \( \Delta\hat{\psi} \) is given as:

\[
\Delta\hat{\theta}_k = \Delta\hat{\psi}_k - T_{\text{gyro}}\hat{b}_k
\]

(6)

In order to obtain incremental inertial attitude \( \hat{C}_{k,k-1} \) from the attitude estimates, following equation is used [7],

\[
\hat{C}_{k,k-1} = 1 - \Delta\hat{\psi}_k^T + \Delta\hat{\theta}_k\Delta\hat{\theta}_k^T - \frac{\|\Delta\hat{\theta}_k\|^2}{2}
\]

(8)

where \( 1 \) denotes the \( 3 \times 3 \) Identity Matrix. This incremental attitude information is used to obtain \( \hat{C}_{k,I} \), the estimate of inertial to body direction cosine matrix, using the relation

\[
\hat{C}_{k,I} = \hat{C}_{k,k-1}\hat{C}_{k-1,I}.
\]

(9)

### B. Steady-state Kalman Gains

In order to obtain the correction equations, we first need to determine the Kalman Filter gains. The gains depend on innovation covariance, error covariance of the process noise and that of the measurement noise. Following the steady-state analysis of [1], the Kalman Gains for each axis are represented using three non-dimensional parameters - dependent on the sensor errors \( \sigma_u, \sigma_v, \sigma_n \) and the correction update interval \( T_{up} \) - characterizing Readout noise, Random-walk noise and Drift angle, respectively, as

\[
S_e = \frac{\sigma_e}{\sigma_n}
\]

(10)

\[
S_u = \frac{T_{up}^{3/2}}{\sigma_u}
\]

(11)

\[
S_v = \frac{T_{up}^{1/2}}{\sigma_v}
\]

(12)

Based on steady-state covariance analysis, steady-state Kalman Filter gains are obtained as

\[
K_{hs} = (\zeta\sigma_n)^{-2}\begin{bmatrix}
P_{\theta\theta}(-) \\
P_{\theta\phi}(-) \\
P_{\phi\phi}(-)
\end{bmatrix} = \begin{bmatrix}
1 - \zeta^{-2}S_u \\
(T_{up})^{-1}S_u \\
(S_e/\zeta)^2
\end{bmatrix}
\]

(13)

where,

\[
\gamma = (1 + S_e^2 + \frac{1}{4}S_u^2 + \frac{1}{48}S_u^2)\frac{1}{2}
\]

\[
\zeta = \gamma + \frac{1}{2}(2\gamma S_u + S_v^2 + \frac{1}{3}S_u^2)\frac{1}{2}
\]

(14)

The steady-state covariances, both pre-update and post-update, are given as, for \( \theta \)

\[
P_{\theta\theta}(-) = (\zeta^2 - 1)\sigma_n^2
\]

(16)

\[
P_{\theta\phi}(-) = (1 - \zeta^{-2})\sigma_n^2
\]

(17)

and for \( b \),

\[
P_{bb}(-) = [\zeta - (1 + S_e^2)\zeta^{-1}\sigma_n T_{up}^{-1/2} + (1/2)\sigma_u^2 T_{up}^{-1/2} - (1/2)\sigma_u^2 T_{up}^{-1/2}]
\]

(18)

\[
P_{bb}(+) = [\zeta - (1 + S_e^2)\zeta^{-1}\sigma_n T_{up}^{-1/2} - (1/2)\sigma_u^2 T_{up}^{-1/2} - (1/2)\sigma_u^2 T_{up}^{-1/2}]
\]

(19)

Although, the formulation for calculating the Kalman Gains along the three axes is same, the value of Kalman Gains for all the three axes will be different, as the value of the attitude sensor white noise (\( \sigma_n \)) is different in all the three axes.

### C. Correction

Correction in the estimates is obtained by utilizing the attitude sensors measurements, which are represented as

\[
C_{n,att,I} = (1 - \nu_{att}) C_{n,I}
\]

(20)

where the vector \( \nu_{att} \) represents the white noise in the horizon sensors and Sun sensor. The standard deviation of the three elements is different as listed in Table II.

To apply the correction using the Kalman Gains expression of representing the measurement residual is required. For the single-axis analysis of [1], the measurement residual based on the scalar attitude measurement \( \theta_{att} \) and \( a \) priori attitude estimate \( \hat{\theta} (-) \) is given as

\[
\theta_{att} - \hat{\theta} (-)
\]

(21)

For the three-axis implementation of the filter an expression of the vector measurement residual is required. In order to obtain a three-axis equivalent of the small angle error we follow the analysis provided in [2]. The small angle residual can be represented for three-axis using the available transformation matrices,

\[
\theta_{att} - \hat{\theta} (-) \Leftrightarrow C_{n,att,I} \hat{C}_{I,n,gyro}
\]

(22)

\[
\approx 1 - \nu_{att/gyro}^\times
\]

(23)

\[
= 1 - \nu_{hs/gyro}^\times
\]

(24)

Hence, \( \nu_{att/gyro}^\times \) characterizes the required measurement residual, and can be obtained in terms of the available matrices \( C_{n,att,I} \) (from attitude measurement) and \( \hat{C}_{I,n,gyro} \) (from estimator),

\[
\nu_{hs/gyro}^\times = 1 - C_{n,att,I} \hat{C}_{I,n,gyro}
\]

(25)

The correction equation in terms of \( \nu_{hs/gyro}^\times \) for attitude,

\[
C_{st/update} = \begin{bmatrix}
(1 - \nu_{x}^\times)\nu_{hs/gyro,x} \\
(1 - \nu_{y}^\times)\nu_{hs/gyro,y} \\
(1 - \nu_{z}^\times)\nu_{hs/gyro,z}
\end{bmatrix}
\]

(26)

\[
\hat{C}_{0,gyro,I}(+) = [1 - \nu_{hs/update}^\times] \hat{C}_{n,gyro,I}
\]

(27)

and bias are given as,

\[
\hat{b}(+) = \hat{b}(-) - \begin{bmatrix}
S_u(x T_{up})^{-1} \nu_{hs/gyro,x} \\
S_u(y) \nu_{hs/gyro,y} \\
S_u(z) \nu_{hs/gyro,z}
\end{bmatrix}
\]

(28)
The filter thus provides estimates of inertial attitude which can be transformed to other frames as per the requirement of the attitude control sub-system. The estimates of bias are used to correct the gyro measurement. Next, we discuss the initialization of the filter and selection of the update interval.

D. Initialization

In order to reduce the filter transients, the filter should be initialized with the best attitude estimate available. On-board this a priori estimate can be obtained from the attitude sensor measurements. These measurements are used to initialize the attitude states of the filter. The drift bias states of the filter should be initialized with the drift bias value as specified in the gyro data sheet or as obtained through ground testing of the gyro. In case prior attitude measurements are not available, then the filter should be initialized with state estimates as zero.

E. Update Interval

In the above formulation, all but one variables influencing Kalman gains are dependent on the sensor characteristic. The parameter \( T_{up} \) which influences Kalman Gains is independent, and can be chosen by the designer. The available values of the \( T_{up} \) will of course be limited because of the sample time of attitude sensors, and computational capability of the on-board computer. Based on steady-state covariance analysis variation of achievable estimate covariance with respect to \( T_{up} \) can be obtained.

Fig. 1 and 2 show the achievable accuracy for different values of update interval. The plots contain both the pre-update and post-update covariance estimates of steady state errors and the standard deviation of white noise of the respective attitude sensor axis. Using the following plots, the update time for our simulations is chosen as 3 seconds as it provides the desired estimation accuracy of \( \sim 0.01 \) degrees at steady-state in all the three axes.

F. Rate Estimation

Estimates of inertial angular velocity of the spacecraft are usually required for control of the satellite, and \( \omega \) is incorporated in the state vector in many filters. However, in our case only gyro drift bias and satellite attitude is estimated. Using the estimates of gyro drift bias the gyro measurement can be corrected for bias error but not for the zero mean random noise. The rate integrating gyro along with the above Kalman filter provides estimate of incremental attitude. Since, the gyro works at a very high rate the incremental angles are very small and will be related to \( \hat{\omega}_{k_f} \) as follows

\[
\Delta \hat{\omega}_{k} = \Delta \hat{\omega}_{k} - T_{gyro} \hat{b}_{k}
\]

\[
\hat{\omega}_{k_f} \approx \frac{\Delta \hat{\omega}_{k}}{T_{gyro}}
\]

However, the above estimate of angular rate still includes the zero mean random noise. A low pass filter, with the following transfer function, is employed to partially eliminate this random noise from \( \hat{\omega} \) estimates

\[
\text{LPF}(s) = \frac{\omega_{lp}}{s + \omega_{lp}}
\]

Selection of \( \omega_{lp} \) will depend on the bandwidth of other subsystems, and should be chosen such that it does not interfere with the controller. As this filter will be implemented on the on-board computer we require digital version of this filter. Using, Tustin’s formula the transfer function is represented in \( z \)-domain

\[
\text{LPF}(z) = \frac{T_{gyro} + T_{gyro}\omega_{lp}z^{-1}}{(T_{gyro}\omega_{lp} + 2) + (T_{gyro}\omega_{lp} - 2)z^{-1}}
\]

The estimate of \( \hat{\omega}_z \) is thus obtained using the low pass filter for each of the three components of the vector \( \hat{\omega}_{k_f} \).

IV. DEVELOPMENT OF SIMULATIONS

In order to validate the attitude determination formulation presented in the previous section, simulations were developed using MATLAB®-Simulink. A controller was included in the simulation so as to observe the overall pointing accuracy obtained by the system. In this section, we present the equations used to simulate the true attitude dynamics, orbit propagation and the controller.
The design of the simulation is kept modular, so that it can be utilized to test different sets of sensors and attitude determination algorithms. Further, all of the above sub-systems were simulated at different rates in order to account for sample time of different sub-systems.

<table>
<thead>
<tr>
<th>Sub-system</th>
<th>Simulation Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>True Dynamics</td>
<td>1000 Hz</td>
</tr>
<tr>
<td>Orbit Propagation</td>
<td>1000 Hz</td>
</tr>
<tr>
<td>Sensors (Gyro)</td>
<td>100 Hz</td>
</tr>
<tr>
<td>Sensors (SS &amp; HS)</td>
<td>1 Hz</td>
</tr>
<tr>
<td>Estimator</td>
<td>100 Hz</td>
</tr>
<tr>
<td>Controller</td>
<td>20 Hz</td>
</tr>
</tbody>
</table>

In our analysis, we consider three frames of reference. The Earth Centered Inertial (ECI) frame is an inertial frame with origin at the Earth’s center. The positive $z$-axis points in the direction of the vernal equinox; the $y$-axis points towards the north pole. The $y$-axis is orthogonal to the other two axes and completes the right handed coordinate system. The Local Vertical Local Horizontal (LVLH) frame describes the current orbit frame of the satellite, and has its origin at the center of mass of the satellite. The $z$-axis points towards the center of the earth (direction of the nadir), the $y$-axis points opposite to the orbit normal and the $x$-axis completes the right handed coordinate system. The instantaneous LVLH frame is used as reference to measure the local attitude of the satellite. Body frame is an orthogonal coordinate system fixed to the satellite body, with origin at the center of mass of the satellite.

A. True Attitude Kinematics and Dynamics

The attitude dynamics of the satellite is described using basic laws of motions for the system, and the set of equations simulated have been listed below. The angular momentum of the satellite and actuator system is expressed as $h$, while the angular momentum of the actuators alone is denoted by $h_a$. $\omega$ denotes the inertial rate of the satellite, while $\omega_a$ represents the angular velocity of the reaction wheels (actuators) with respect to the satellite. $g_{con}$ and $g_{dist}$ denote the control and disturbance torque, respectively.

$$\mathbf{I}\dot{\omega} = -\omega \times h + g_{con} + g_{dist}$$  \hspace{1cm} (33)$$

$$\dot{h_a} = -g_{con}$$ \hspace{1cm} (34)$$

where,

$$\begin{bmatrix} h_{ax} \\ h_{ay} \\ h_{az} \end{bmatrix} = \begin{bmatrix} I_s & 0 & 0 \\ 0 & I_s & 0 \\ 0 & 0 & I_s \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} + \begin{bmatrix} \omega_{ax} \\ \omega_{ay} \\ \omega_{az} \end{bmatrix}$$

$$h_a = I_s(\omega + \omega_a)$$

$$h = I(\omega + \omega_a)$$

The above equations are moment balance equation of satellite and the actuator, respectively. Euler angles with respect to LVLH are denoted as $[\phi \ \theta \ \psi]^T$. The kinematics of the satellite is described by relating the inertial angular rates $\omega_{BL}^B$ to the derivatives of Euler angles (with respect to LVLH).

$$S^{-1} \omega_{BL}^B = S^{-1} (\omega_{BL}^B - C_{BL} \omega_{LI}^L)$$ \hspace{1cm} (35)$$

$$S^{-1} = \frac{1}{\cos \theta} \begin{bmatrix} \cos \theta & \sin \theta \sin \phi & \sin \theta \cos \phi \\ 0 & \cos \theta \cos \phi & -\cos \theta \sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix}$$ \hspace{1cm} (36)$$

For the purpose of simulation, $\mathbf{I}$ the moments of inertia are taken as

$$\mathbf{I} = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 10 \end{bmatrix} \text{ kg m}^2$$
B. Orbit Propagation

Orbit propagation is required to determine the position of the satellite in space, which in turn determines the transformation from ECI frame to LVLH frame. A simplified orbit propagator is used to obtain this transformation. The position is described in terms of the six orbital parameters. The right ascension of the ascending node changes due to the precession of the orbit, however this change is relatively small for the simulation time period, and hence the ascending node is taken constant for the purpose of our simulations. A circular low Earth orbit with altitude equal to 500 km is chosen for simulation. Other parameters of the orbit are tabulated as follows:

<table>
<thead>
<tr>
<th>Orbital Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semi-major axis, a</td>
<td>$R_{\text{earth}} + 500$</td>
<td>km</td>
</tr>
<tr>
<td>Eccentricity, e</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Inclination, i</td>
<td>95.3</td>
<td>deg</td>
</tr>
<tr>
<td>Ascending Node, $\Omega$</td>
<td>90</td>
<td>deg</td>
</tr>
<tr>
<td>Argument of perigee, $\omega_{\text{per}}$</td>
<td>0</td>
<td>deg</td>
</tr>
<tr>
<td>Orbital rate, $n$</td>
<td>0.0011</td>
<td>rad/s</td>
</tr>
<tr>
<td>True anomaly, $\nu_{\text{anomaly}}$</td>
<td>$n^*(\text{time})$</td>
<td>rad</td>
</tr>
</tbody>
</table>

The transformation matrix from ECI frame to LVLH frame, $C_{LI}$, based on the above parameters is given as,

$$C_{LI} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix} C_{OI}$$  \( (37) \)

$$C_{OI} = \begin{bmatrix} c\Omega & s\Omega & 0 \\ -su & cu & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\Omega & s\Omega \\ 0 & -si & ci \end{bmatrix}$$

where, $u = \omega_{\text{per}} + \nu_{\text{anomaly}}$

C. Controller

A controller is required for the purpose of simulation, in order to observe the closed loop performance of the attitude estimator. Several controllers both linear (PD, PID) and non-linear exist in the literature for satellite applications. The choice of controller depends on the design specification and the type of actuators used.

<table>
<thead>
<tr>
<th>TABLE V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Controller Parameters</td>
</tr>
<tr>
<td>$\delta$</td>
</tr>
<tr>
<td>$\zeta$</td>
</tr>
<tr>
<td>$\omega_n$</td>
</tr>
</tbody>
</table>

Here, we use a basic PID controller for our simulations. The gains of the PID controller were selected to obtain the desired damping ratio ($\delta$) of 0.707, and natural frequency ($\omega_n$) corresponding to a time period of half minute or (0.5)/60 seconds. The parameter $\delta$ influences the integral gain, and helps to eliminate the steady-state error of the controller. The values of the normalized controller gains, for the listed specifications, are listed in Table VI.

Based on the above values of controller gains the control torque vector is determined.

$$g_{\text{con}} = I \left( a_p \theta_{rr} + a_\nu \nu_{err} \right) + a_d \omega_{err}$$  \( (38) \)

$$\theta_{err} = \theta_{\text{com}} - \hat{\theta}$$  \( (39) \)

$$\omega_{err} = \omega_{\text{com}} - \hat{\omega}$$  \( (40) \)

Lastly, for all the simulation results presented the commanded attitude and angular rate are given in Table VII.

| TABLE VII |
| Commanded Attitude and Rate |
| $\phi$ | $\theta$ | $\psi$ |
| 0 | 0 | 0 |
| $\omega_x$ | $\omega_y$ | $\omega_z$ |
| 0 | $-n$ | 0 |

In order to apply the control torque as calculated above hardware (actuator) is required. As was with the controllers, various actuators too exist for satellite control. For our simulation, we assume that three orthogonal reaction wheels are used to apply the requisite control torque along the three body axes. The applied control action does not vary continuously, but changes after every 0.05 seconds, i.e. at a rate of 20 Hz.

V. Simulation Results

In this section, we present the output of the attitude determination method using the described simulation. Firstly, the simulations are validated by checking the system performance without any estimator. Next, the performance of the filter is presented with the specified sensor errors. Lastly, performance of filter subject to different conditions, such as, eclipse, higher $T_{up}$ is discussed.

| TABLE VIII |
| Initial Attitude and Rate Error |
| $\phi$ | $\theta$ | $\psi$ |
| 2 | 3 | 4 |

All the simulations have been initialized by providing initial disturbances in the attitude and rate, as listed in Table VIII. The controller tries to reduce these deviation and to bring the system to the desired state. No disturbance torques are considered in the simulations. It should be noted that, in our implementation the correction from the attitude sensors is applied to the filter as and when a measurement is received, and the algorithm does not need to wait till information is available for all the three axis.
A. Validation

As there are no disturbance torques, the total angular momentum of the spacecraft and the reaction wheel should remain conserved in the inertial (ECI) frame. The angular momentum should remain conserved, whether, the estimator and/or controller is used or not. Hence, for the validation of each simulation run, change in angular momentum is observed. Variation in angular momentum, caused due to the solver, is observed to be lower than 0.01%. Fig. 4 shows the plot of angular momentum for a typical simulation run.

B. Controller Performance

In order to observe performance of the controller in isolation, behavior of the satellite without any estimator is observed. This is done to check the control sub-system, its transients and steady state performance.

C. Kalman Filter Performance

The results of the three-axis attitude determination algorithm are now presented. The parameters update interval $T_{up}$ and the low pass filter cut-off frequency $\omega_{lp}$ are taken as 3 seconds and 1 Hz respectively. The Kalman filter is initialized with measurements from attitude sensors (Sec. III-D). Thus estimation errors in attitude are similar in magnitude to the attitude sensor noise, and for our simulation are taken as specified in Table IX.

The magnitude of initial errors in state estimates depends on the accuracy of reference sensors, hence the initial attitude estimation error is of the order of 0.1 degrees. However, it should be noted that this estimation error is present along with the initial pointing error, which is larger in magnitude as specified in Table VIII. If no attitude reference information is available a priori, then the filter will be initialized with state estimates as zero, resulting in larger initial estimation error.

Fig. 7, compares the measured attitude by the attitude sensors (measurement error) and estimated attitude by the filter (estimation error) with respect to the true attitude, for one component of the attitude ($\phi$). As can be seen from Fig. 8, the attitude estimates converge to the true value within approximately one third of the orbital period. The estimation errors for the gyro drift-rate bias (Fig. 9) do not decay to zero, however they converge to a finite steady-state value.
remained bounded for the period of simulation of 1800 seconds (which approximately corresponds to the eclipse phase of a low Earth orbit). After the eclipse phase, when complete attitude information was available, the filter resumed function and reduced estimation errors in all the three axes.

### F. Low Pass Filter

Lastly, we demonstrate the improvement obtained due to the use of a Low Pass Filter. As angular rate is not an element of the state vector of the filter, white noise persists in the estimate of angular rate obtained using the estimate of bias drift. The low pass filter removes this noise partially and improves the estimates of the $\omega$. The applications of the low pass filter strongly affect the calculation of control torque, which is directly dependent on the estimate of $\omega$. The low pass filter improves the applied control torque by removing the unwanted high frequency changes in the torque, which might have adverse effects on both the actuator hardware as well as the performance of the attitude determination algorithm.

### D. Variation with Update Time

For the above set of simulations, update time was taken to be 3 seconds. Simulations with a larger update time also yielded a reduction in estimation error although some reduction in performance. As predicted by the analysis in section III-E, with increase in update time the steady state covariance increases, due to which the achievable pointing accuracy reduces. Also, the initial transients in the filter take longer to decay. Lastly, the measurement update from attitude sensors produce a more visible correction in the estimation errors, specially when the correction intervals were larger than 100 seconds.

### E. Partial update

The attitude sensors generally provide complete attitude information. However, during flight some cases may arise when only partial attitude information might be available. This may happen for instance during the eclipse phase of the satellite orbit when the sun sensors do not function. Performance of the filter for this case was also examined, with only partial attitude information along roll and pitch available, and yaw information unavailable. The estimates along the yaw-axis did not converge, however they still
G. Recursive gain Kalman Filter

Simulation with attitude determination using the conventional, recursive gain Kalman Filter were done, in order to compare the performance of the steady-state Kalman filter. The recursive gain Kalman Filter provides optimal gains throughout and not just at steady state; thus, causing reduction in transient period by around 200 seconds to achieve the corresponding steady-state covariance, however, at a much higher computational cost. Hence, depending on the mission requirements and available computation power, a choice can be made whether to use the recursive gain Kalman Filter or its steady-state version.

With the help of simulations, we demonstrate that the steady-state Kalman Filter is able to achieve desired estimation accuracy. The attitude estimation errors converged to desired steady state value for update interval of 3 seconds. Drift bias estimation errors did not decay to zero, however, stabilized to an acceptable constant steady state error. Hence, based on the on-board computational capabilities of the satellite, the steady-state Kalman Filter may be a favorable choice.

Use of the steady-state Kalman Filter has obvious computational advantages for on-board implementation; hence, in order to further confirm the above conclusion, advanced sensor error models as well as realistic orbit propagation should be included in future analysis. Furthermore, to confirm that the controller along with the estimator will eventually lead to stability of the equilibrium point of the system, stability analysis needs to be performed. However, the present analysis and simulation, despite the aforesaid simplifications, illustrate the use of steady-state Kalman Filter as a possible alternative to the conventional recursive gain Kalman Filter, for attitude estimation of satellites.

VI. Conclusion and Comments

Attitude determination simulations for a low Earth orbiting satellite were developed. A PID controller was used to test the closed loop performance of the estimator. A three-axis steady-state Kalman Filter was implemented and tested.

References